

XXVIII. *On the Nature of the Forces concerned in producing the greater Magnetic Disturbances.* By BALFOUR STEWART, M.A., F.R.S.

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1. IN a previous communication submitted to the Royal Society on June 28th, 1861, and since published in their Transactions, I ventured to make a suggestion regarding the nature of that connexion which subsists between magnetic disturbances, earth-currents, and auroras.

In this hypothesis the earth was viewed as similar to the soft iron core of a RUHM-KORFF'S machine, in which a primary disturbing current was supposed to induce magnetism. Earth-currents and auroras, on the other hand, were viewed as induced or secondary currents, caused by the small but abrupt changes which are constantly taking place in the strength of the primary disturbing current, these changes being very much heightened in effect by the action of the iron core, that is to say, of the earth.

2. These small and rapid changes are very observable in the photographic traces given by the Kew magnetographs during a time of disturbance. At such a time the curves for all the three elements invariably present a serrated appearance, which, when the disturbing cause is very powerful, is magnified into a succession of sharp peaks and hollows, and these exhibit a frequency of change which makes them comparable in this respect with earth-currents and auroras.

These peaks and hollows are therefore regarded by this hypothesis as denoting the changes which take place in the action of the primary disturbing current upon the needle through the intervention of the earth's magnetism, or, since the existence of a primary current is really unnecessary as an explanation, we may dispense with it altogether *, and regard these photographic peaks and hollows as simply representing the changes which take place in the magnetism of the earth, without speculating upon the cause of such changes.

3. In the paper already alluded to, in discussing the great magnetic storm extending from August 28 to September 7, 1859, I endeavoured to show that the first effect of the superimposed disturbing force was to diminish both elements of the earth's magnetism during a period of about six hours. This grand wave, constituting the great body of force, could not, I remarked, be supposed to be due to any combination of earth-currents of which the period is only a few minutes. Under these circumstances it would seem to be the most natural supposition to regard the peaks and hollows, which occur as it were on the surface of the great disturbance wave, as representing rapid changes in the in-

* This was suggested to me by Professor TYNDALL.

tensity of the disturbing force, and therefore not as phenomena caused by earth-currents, since the great body of the disturbance of which they represent the changes cannot be due to any such cause. Auroras and earth-currents being thus removed out of the list of causes, might then be supposed to follow rather as effects of these magnetic changes, on the principle of voltaic induction already mentioned. As, however, there seems to be a tendency to regard the motions of the magnetic needle as due to the direct action of earth-currents, it may be desirable here to inquire whether this view of the origin of these small and rapid disturbances is capable of being accepted as a tenable hypothesis.

4. It is almost unnecessary to remark that the hypothesis which asserts that earth-currents are not the cause of the small and rapid magnetic disturbances, does not assert that such currents have absolutely no influence upon the needle, since we know that a current must always act upon a needle; but it maintains that these currents are induced and secondary phenomena, and therefore represent only a small fraction of the whole force in operation, and react on the magnet only to a very limited extent.

5. The interesting observations on earth-currents made under the direction of Mr. C. V. WALKER, and recorded by him in a paper recently published in the Transactions of the Royal Society, appear to furnish grounds for an hypothesis regarding these phenomena.

By reference to a map appended to his paper, it appears that a line drawn from the Ramsgate to the Margate telegraph station proceeds in a direction $12^{\circ} 50'$ W. of true north; or, assuming $21^{\circ} 30'$ W. as the approximate magnetic declination, this line proceeds in a direction $8^{\circ} 40'$ E. of magnetic north. In like manner, a line drawn from the Ashford to the Ramsgate telegraph station proceeds in a direction $83^{\circ} 50'$ E. of magnetic north; also a line drawn from the Ashford to the Margate station in a direction $77^{\circ} 42'$ E. of magnetic north.

By Mr. WALKER'S observations from August 29 to September 2, 1859, and also from August 8 to August 10, 1860, it would appear that a current proceeding from Margate to Ramsgate was always simultaneous with one proceeding from Ramsgate to Ashford, and with one proceeding from Margate to Ashford; that is to say, simultaneous currents proceeding along these lines were either all north or all south currents.

Let us endeavour to ascertain how far this agrees with the behaviour of the magnetographs at Kew during these two disturbances on the hypothesis of direct action, assuming also with Mr. WALKER that earth-currents are derived from a stream of electricity which is drifting across the country.

Conceive now a current resolved into two components, the one proceeding in the magnetic meridian, and the other in a direction perpendicular to it; and let us agree to consider as positive those currents flowing from magnetic north to south, and from magnetic east to west. Of these two components, the first only will affect the freely suspended declination magnet, a positive current tending to increase the westerly declination; while the latter only will affect the horizontal-force magnet which has been twisted into a position at right angles to the magnetic meridian, a positive current increasing the horizontal force.

Let x be the value of the north and south, and y that of the east and west component of a current as deduced from its effects upon the magnetographs.

This will give on the Margate and Ramsgate line a current represented by

$$A \{x \cos 8^\circ 40' + y \sin 8^\circ 40'\}, \dots \dots \dots (1.)$$

while for the Ramsgate and Ashford line we shall have

$$B \{x \cos 83^\circ 50' + y \sin 83^\circ 50'\}, \dots \dots \dots (2.)$$

and for the Margate and Ashford line

$$C \{x \cos 77^\circ 42' + y \sin 77^\circ 42'\}, \dots \dots \dots (3.)$$

where A, B, C are positive constants depending upon the nature of the soil and strata along the various lines.

Now in the disturbance of 8th to 10th August 1860, the declination and horizontal force were increased simultaneously, or they were diminished simultaneously; hence x and y have the same sign, and the expressions within brackets are either all positive or all negative, that is to say, the currents between these lines should have been either all north or all south simultaneously. This was observed to be the case. In this instance, therefore, there is nothing to contradict the hypothesis which makes earth-currents the cause of magnetic disturbances.

Let us now examine the disturbance of August 29 to September 2, 1859. Here the horizontal force was increased when the declination was diminished, and *vice versa*; and if we represent by unity the current influential in disturbing the former element, that disturbing the latter is most probably represented by $-.46$ (see Table V.).

Hence (1.), (2.), (3.) become

$$\begin{aligned} (1) &= A \{-.46 \cos 8^\circ 40' + \sin 8^\circ 40'\} = -.304 A, \\ (2) &= B \{-.46 \cos 83^\circ 50' + \sin 83^\circ 50'\} = +.945 B, \\ (3) &= C \{-.46 \cos 77^\circ 42' + \sin 77^\circ 42'\} = +.879 C. \end{aligned}$$

We thus see that in this disturbance, for the theory to hold good, currents of one name should have traversed the line between Margate and Ramsgate, while currents of the opposite name traversed the other two lines; but this is not in accordance with the observations made, which showed that currents of the same name traversed the three lines simultaneously. In one word, the proof against the theory of the direct action of earth-currents in causing disturbances may be summed up by saying that, while the magnetic disturbances of September 1859 and August 1860 were of opposite character, the corresponding earth-currents, as determined by these three telegraphic lines, were of the same character.

6. At first view it would almost appear as if this fact, which seems conclusive against earth-currents (at least those earth-currents observed in this country) being the direct and main cause of disturbances, were equally conclusive against their being induced currents, since the effect of such upon the telegraph needle might be supposed opposite in character to that which would be produced by such currents as would directly cause magnetic disturbances, the two effects being antithetically and therefore definitely related to each other.

But this difficulty will disappear when we reflect that earth-currents may be twisted in passing over the surface of our globe. Although, therefore, we may not find that oppositeness of character which we naturally associate with induced currents, yet the study of particular earth-currents in connexion with simultaneous magnetic disturbances may perhaps serve to throw some light upon the nature of the former.

7. The following is brought forward as an instance of this. On September 1st, 11.20 A.M. G. M. T., a *strong southerly* current was noticed on the Ashford and Margate line which lasted till 11.26 A.M., while from 11.28 A.M. to 11.35 A.M. a *slight northerly* current was observed on the same line.

If we take into account a slight delay in the starting of the Kew curves (equal to about 2 minutes) due to the slackness of the toothed wheels which drive the cylinders, we find the first recorded appearance of the south current to be, in point of time, $1\frac{1}{2}$ minute behind that of a very abrupt disturbance which affected all the magnetic elements at Kew, and which occurred simultaneously with the outbreak of a curious phenomenon on the sun's disk. Whatever be the cause of this apparent difference in time, I do not think that it argues any want of simultaneity between the magnetic disturbance and the earth-current, and we may safely suppose that the first outbreak of the south current was really simultaneous with the commencement of the disturbance at Kew.

The magnets at Kew were affected in the following manner. The westerly declination was at first rapidly increased; but soon the disturbing force attained its maximum, and then gradually diminished, until ultimately the needle attained nearly the same position which it had before the disturbance. The horizontal force was at first rapidly diminished, then as the disturbing force died away it also gradually came back to its previous value. A direct current which would cause a disturbance of this nature would be one at first rapidly increasing, then gradually diminishing, but always preserving the same name, whereas the induced effect of such a disturbance would be at first a strong current in one direction, and afterwards a weak current in the opposite direction. The latter character agrees very well with that of the currents between Ashford and Margate, the former character not in the least. Judging therefore from this disturbance, and, though only a single example, it appears to be an unexceptionable one, it would seem that earth-currents do not cause magnetic disturbances, but are rather the induced currents which the latter give rise to.

8. I shall now endeavour to show that we have grounds for supposing the magneto-graph peaks and hollows to be due to changes in the value of the disturbing force.

The action of any such disturbing force is of a twofold nature.

1°. It raises a curve above its normal position, or depresses it below the same, visibly and for a somewhat lengthened period of time.

2°. The curve which represents this definite action of the force is at the same time studded with sharp peaks and hollows.

Now if the second of these two effects denotes the changes which occur in the force

producing the first, the peaks and hollows should be similar in character to the general effects of the force. The following example will explain what is meant.

If we have a force which simultaneously and for a lengthened period of time raises up the curves of all the three elements nearer to the top of the paper, and if the peaks and hollows denote changes taking place in the intensity of this force, then all the three elements should exhibit peaks at once, or hollows at once. But if, on the other hand, the general appearance of the curves represents the declination as raised up while both the other elements are depressed, then a peak in the former should correspond to a hollow in the latter two.

9. I requested Mr. CHAMBERS, Magnetical Assistant at Kew, to note his impression of the general appearance of the disturbance curves of most importance for the years 1858, 1859, and 1860; and the following Table is the result. The sign + means that the curve is raised towards the top of the paper, the sign - means that it is lowered towards the bottom.

TABLE I.—General appearance of Disturbance Curves.

Date.	Character of disturbance.		
	Declination.	Horizontal force.	Vertical force.
1858. March 28—29	—	+	+
April 9—10	—	—	—
June 23—24	+	—	—
July 5—6	0	—	—
October 27—28.....	indefinite.	indefinite.	+
December 4—5	+	+	—
1859. February 9—10.....	+	0	—
February 9—10.....	+	0	+
February 26—27	—	indefinite.	—
April 21—22	—	—	—
April 21—22	+	+	+
April 29—30	—	—	—
May 19—20	—	—	—
June 8—9	—	—	—
June 8—9	indefinite.	+	+
July 11—12	0	—	—
July 18—19	—	0	—
August 28—29	—	+	+
September 1—2.....	—	+	+
September 2—3.....	—	—	—
September 3—4.....	—	—	—
October 12—13.....	—	—	indefinite.
October 17—18.....	+	+	+
October 17—18.....	—	+	+
October 18—19.....	—	—	—
October 18—19.....	—	+	+
December 13—14	—	—	—
December 13—14	indefinite.	0	—
December 14—15	—	0	+
1860. March 28—29	—	—	+
March 29—30	—	—	—
April 9—10	—	—	—
April 9—10	+	+	+
July 4—5	+	+	+
July 5—6	—	—	—
August 6—7	indefinite.	—	0
August 7—8	0	—	—
August 8—9	—	—	—
August 9—10	0	—	—
August 12—13	—	—	—
August 12—13	—	+	+
September 7—8	—	—	—

It thus appears that there are twenty-two cases in which the declination is raised or lowered along with the horizontal force, and only seven cases of an opposite description. Also there are twenty-two cases in which the declination is raised or lowered along with the vertical force, and only eleven cases of an opposite description. Finally, there are thirty-one cases in which both forces are raised or lowered together, and only two cases of an opposite description.

There is therefore a decided tendency in the curves of all the elements to be raised or lowered simultaneously; but this tendency is stronger between the horizontal and vertical-force curves than between either of these and the declination. It may at the same time be affirmed that, with the exception of the disturbance of August—September 1859, there is no very prominent case in which the three elements do not rise or fall together.

10. Having thus recorded the general appearance of the curves, I shall now give the result of the examination of the simultaneous peaks and hollows; but it will first be necessary to state the method in which this has been conducted.

Each curve has a zero-line, or line of abscissæ, along which the times are reckoned; so that if we wish to find the time corresponding to any point in the curve, we have merely to measure its abscissa, the commencement of the zero-line (denoting the moment at which the instrument was started) being the origin.

In like manner, it is equally easy to find the point of the curve corresponding to any given time.

The accuracy of this process depends, however, it will be seen, on the assumption that the time-scale is constant for the different portions of a curve; and the following is a proof that this is strictly true.

It will be noticed shortly that this system of measurement has brought out a remarkable correspondence between the peaks and hollows of the horizontal force and those of the vertical force, which may be said to have failed in no one instance. This may be received as sufficient evidence, not only of the physical fact brought to light, but also of the constancy of the time-scale, in absence of which no such phenomenon could have been observed.

11. The following Table exhibits the results derived from comparing together the peaks and hollows of the declination, with those occurring at the same instant of time in the horizontal force.

When peak and peak occur together, or hollow and hollow, this is termed a correspondence, the reverse a non-correspondence.

TABLE II.—Exhibiting the connexion between the small movements of the declination and those of the horizontal force in the various disturbances.

Date of disturbance.	Number of correspondences.	Number of non-correspondences.	Doubtful.
1858. March 12—16	18	1	5
April 9—12.....	24	0	5
June 22—24	13	0	1
October 27—29	15	0	0
December 4—6	12	1	1
1859. February 8—10	13	2	1
February 26—28	19	0	2
April 21—23	14	0	1
April 29—May 1	15	0	1
May 19—21	14	0	0
June 8—10.....	13	0	1
July 11—13	10	0	5
October 12—14	8	1	2
October 17—19	19	0	2
December 13—15	9	0	1
1860. March 26—30	29	0	2
April 9—11	22	0	5
April 13—15	15	1	1
June 29—July 6.....	28	0	3
August 6—13	65	12	46
September 7—8	7	0	0
Sums...	382	18	85

It will be seen from this Table that, in the great majority of cases, a peak in the declination corresponds to a peak in the horizontal force, and a hollow in the one to a hollow in the other. It is proper to mention that the peaks and hollows here observed are those which represent sudden changes of short duration; for if we take for comparison some prominent peak or hollow in the one curve having a long period, we shall be much less certain of finding a corresponding phenomenon in the other.

12. When the peaks and hollows of the horizontal force and those of the vertical force are compared together, the result is that a peak invariably corresponds to a peak, and a hollow to a hollow; and even when large prominences of long duration are taken, the correspondence between the two curves is very remarkable.

The same connexion, therefore, which subsists between the sudden movements of the declination and those of the horizontal force, holds still more strikingly between those of the two forces.

13. The disturbance of August—September 1859 has been purposely left out of Table II.; in the following Table this great disturbance has been broken up into parts, for each of which the behaviour of the peaks and hollows is compared with the general appearance of the curve.

TABLE III.—Great disturbance extending from August 28 to September 7, 1859. The behaviour of peaks and hollows compared with the general appearance of the curves.

Time.	Declination movements compared with those of the horizontal force.			General character of the disturbance.
	Correspondence.	Non-correspondence.	Doubtful.	
1859. August 28, 10 A.M. to 9 P.M.	10	0	1	The great disturbance had not yet commenced.
August 28, 10.30 P.M. to August 29, 9.52 A.M.	0	6	0	The great disturbance had now commenced depressing the declination and raising both elements of the force.
September 2, 4.50 A.M. to September 3, 11.50 A.M.	0	20	2	A second great disturbance commenced about September 2, 4.50 A.M., depressing the declination and raising both elements of the force, which changed into or was succeeded by one which seemed to impress all the elements in the same manner.
September 3, 1.40 P.M. to September 4, 10.30 A.M.	9	13	3	
September 4, 12.30 P.M. to September 6, 9.30 A.M.	16	3	7	

14. Before discussing the results in this Table, I ought to mention that in this great disturbance the rapid nature of the motion makes the comparison of simultaneous peaks and hollows a matter of some little uncertainty. On the other hand, these comparisons have been much facilitated by the exceedingly good definition which the labours of the late Mr. WELSH secured for the Kew curves, without which, indeed, an investigation of this nature would have been impossible. On the whole, I am well persuaded that the results in Table III. represent the truth.

15. To recapitulate. It appears from Table I. that, studying merely the general result of a disturbance, there is a decided tendency to raise or lower the curves of all the elements simultaneously, this being stronger between the horizontal and the vertical-force curves than between either of these and the declination.

From Table II. it appears that, if we leave out of account the great disturbance of August—September 1859, a peak of declination corresponds to a peak of horizontal force, and a hollow of the one to a hollow of the other, while the same correspondence holds still more strongly between the horizontal and vertical forces.

Again, from Table III. it appears that the great disturbance of August—September 1859 may be broken up into two. In one of these the general result was to lower the declination and raise both elements of the force, while in the second the result was to raise or lower all the three curves simultaneously. It also appears that, while the first of these disturbances prevailed, a declination hollow corresponded to a peak of either force, and that, on the other hand, while the second prevailed, a declination hollow corresponded to a hollow of either force.

This very marked correspondence between the behaviour of the peaks and hollows and that of the general disturbing force *in all cases*, leaves, I think, little doubt that the former represent sharp and sudden changes in the intensity of the latter.

16. Let us now endeavour to ascertain if any use may be made of this fact to throw light on the nature of the forces concerned in producing disturbances.

17. Two distinct suppositions may be made regarding the nature and mode of action of disturbing forces.

1°. We may suppose that forces of every imaginable variety of character are concerned together in producing disturbances.

2°. We may suppose a disturbance occasioned by one or more groups of forces the elements of which are bound together by a certain law.

With respect to the first of these hypotheses, it is refuted by the discussion of disturbing forces given by General SABINE for the different Colonial Observatories and for Kew, as well as by the results in Tables I., II., and III. The second hypothesis must, therefore, represent the mode of action of the forces concerned.

18. And, first of all, it may safely be affirmed that no disturbance of any magnitude is due to the action of a single force, merely varying in amount; for if this were the case, the distance at any moment of a point in the curve of one of the elements from its normal position should bear throughout a disturbance an invariable proportion to the distance of a corresponding point in the curve of another of the elements from its normal; but this is by no means true.

Since, therefore, a disturbance is not a phenomenon due to the action of a single force, and since at the same time it does not represent the action of a number of different forces promiscuously huddled together, it becomes a question of interest to ask ourselves how we may find the elementary forces concerned.

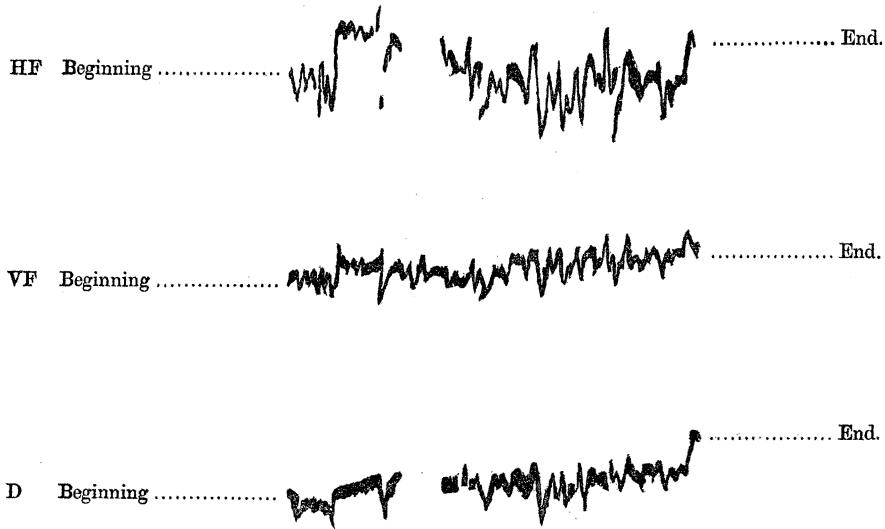
19. A little consideration will show that this is likely to be obtained by the study of small and rapid changes of force. For if several forces are at work, it is unlikely that at the same moment a sudden change should take place in all; there is thus a high probability that a sudden and rapid change is a change in one of the elementary forces concerned, and which will therefore enable us to determine the nature of that force. Even if the change is not a very abrupt one, provided that we confine ourselves to such peaks and hollows as present a similar appearance for all the curves, we may be satisfied that we are observing changes taking place in one only of the elementary disturbing forces; for it is inconceivable that two or more independent forces changing independently should produce similar appearances in all the three curves. This may be illustrated by an example. Suppose, for instance, a disturbance takes place in declination which at the end of one minute has raised the curve one-tenth of an inch, while at the end of the second minute it has fallen again to its original level. Suppose also that a change, precisely the same in nature and amount, takes place in the horizontal force, while in the vertical force the change is only one-twentieth of an inch. Here then we have in our curves three isosceles triangles which, although the last is not, strictly speaking, similar to the other two, yet all three convey the idea of only one force acting. Suppose now another force to have been superimposed which affected the declination very much more than either of the force-components, but which did not begin to work until the end of the first minute. The result would be that the declination peak would be much further

from an isosceles triangle than that of either of the forces; we might even suppose the the descending limb of the declination peak to become level, or even to ascend in consequence of the action of this second force.

The following is a more precise definition of what is requisite before we can refer the action to a pure elementary force.

Let it be supposed that we are comparing together portions of each of the three curves, commencing at the same time, then it may be asserted that these portions are due to the action of a single disturbing force when (the origin for each being the commencement of the portion of the curve under consideration) for equal abscissæ the ordinates of the one curve always bear a definite proportion to those of the other.

20. This is illustrated by the following tracings, which exhibit the movements of the three curves on August 11, 1860, from 2.34 P.M. to 5.26 P.M.



21. From all this we perceive that, in order to find what are the pure elementary forces at work, we must select suitable peaks and hollows, and measure these accurately.

The following Table exhibits the results obtained by comparing such peaks and hollows of the horizontal force with those of the vertical force, no reference being made to the declination.

TABLE IV.—Comparison in length of the Horizontal-force changes with those of the Vertical Force.

Date.	Horizontal-force change (vertical-force change=unity in each instance).	
	Actual measurements.	Mean.
1858. March 2—3.....	2.4, 2.1, 2.3, 2.0	2.2
March 12—16	1.9, 1.9, 2.1, 1.8, 1.9, 2.3, 2.0, 2.1	2.0
March 27—29	2.3, 1.9, 2.0	2.1
April 9—10.....	1.9, 2.1, 2.0	2.0
June 23—24	2.3, 2.1, 2.0	2.1
July 5—6	2.1	2.1
1859. February 9—10	1.9, 2.2, 2.2	2.1
February 27—28	1.9, 2.1	2.0
May 19—21	2.0, 1.9, 2.2, 2.1	2.0
June 8—9	{ 2.2, 2.0, 2.0, 2.2, 2.2, 2.0, 2.0	} 2.0
July 11—13	{ 2.1, 1.9, 2.1, 1.9, 1.9, 2.0, 2.1	
August 28—September 7.	{ 2.0, 2.1, 2.1, 1.9, 2.0, 2.2	} 2.0
	{ 1.8, 2.0, 2.0, 2.0, 1.9, 1.6, 2.0	
	{ 2.1, 2.2, 2.2, 2.2, 2.2, 2.0, 2.2	
	{ 2.3, 1.6, 2.0, 2.0, 2.0, 2.1, 1.8, 1.7	
October 12—18	{ 2.0, 1.9, 1.9, 2.1, 1.9, 2.2, 1.9, 1.7, 2.1	} 2.0
December 13—15	{ 2.0, 2.1, 2.0, 2.1, 2.0, 1.9, 2.0, 2.2, 2.0	
1860. March 26—29	{ 2.0, 2.0, 2.0	} 2.0
April 9—15	{ 2.3, 1.9, 2.0, 1.9, 1.6, 1.8, 1.9, 1.6	
July 4—6	{ 2.2, 2.0	} 1.9
August 6—12.....	{ 2.1, 2.3, 2.0, 2.1, 1.9, 2.0, 1.8, 2.0	
August 6—12.....	{ 1.8, 2.1	} 2.0
	{ 2.1, 1.9, 2.1, 2.0, 2.0, 2.2, 1.7, 2.3, 2.0	
	{ 2.1, 2.0, 2.1, 1.9, 2.1, 1.9, 1.9, 2.2	
August 6—12.....	{ 1.9, 1.9, 2.0, 2.1, 2.0, 2.1, 2.0, 1.9	} 2.0
	{ 2.0, 2.0, 1.9, 2.2, 2.1, 1.9, 2.0, 1.9	
	{ 2.0, 2.2, 1.9, 2.1, 2.0, 2.0	

In this Table the actual measurements have been exhibited in order to afford an idea of the accuracy with which the proportion holds, without attempting to estimate the probable error.

22. In endeavouring to frame a similar Table between the horizontal force and declination, a curious fact presented itself.

Although it is comparatively easy to find changes in the vertical force which are similar to corresponding changes in the horizontal force, yet it is much more difficult to find similar corresponding changes in the horizontal force and declination. Indeed, for many of the cases recorded in the above Table, the declination-change would not be similar to that of either force. It thus appears that, even in cases which do not indicate the action of a pure elementary disturbance, the horizontal-force change preserves an almost invariable relation to that of the vertical force*.

23. The following would appear to be the explanation of this.

Whatever be the nature of an elementary disturbing force, its effect upon the hori-

* This proves, in addition to a physical fact, that these magnetographs are capable of recording with precision the slightest change in either element of the earth's force.

zontal component of the earth's magnetism bears always a nearly invariable proportion to its effect upon the vertical component of the same. In this case it is clear that two or more disturbing forces superimposed would, as far as regards the horizontal and vertical-force changes, behave almost in the same manner as a pure elementary disturbance. This also explains why it is almost impossible to find a peak in the one component of the force corresponding to a hollow in the other.

24. This curious fact may likewise be stated in the following language :—

Whatever be the nature of the disturbing force at work, its resolved portion, which acts in the plane of the magnetic meridian at Kew, is always in nearly the same line.

25. It is very easy to find the direction of this line.

A disturbance of the horizontal force, as it appears in the curve, is, we have seen, very nearly double that of the vertical force. Now an inch in the ordinate of the horizontal-force curve denotes a change amounting to nearly .010 of the whole force, while for the vertical-force curve it represents a change of .0025 of the whole ; and increasing ordinates denote decreasing force for both elements. Also the absolute value of the horizontal force is 3.8, while that of the vertical force is 9.6.

Hence if we represent by 38 the value of the horizontal component of the disturbing force in the plane of the magnetic meridian, that of the vertical component will be denoted by 12, and the dip of the whole force acting in this plane will be 17°5 nearly.

26. From what has been already mentioned (art. 22), it may be inferred that it is difficult to obtain similar corresponding changes for all the elements together. The following Table exhibits those instances in which this has been accomplished ; but the results can only be regarded as very rough approximations.

In some cases, where the vertical-force disturbance was very small, it was not attempted to measure it.

TABLE V.—Similar corresponding changes for all the elements.

Date.	Greenwich Mean Time.		Character and size of change (vertical-force change=unity in each instance).			
			Declination.	Horizontal force.	Vertical force.	
1858.	h	m				
March 14.....	2	19	P.M.	rise after =2.1	rise after =1.9	rise after =1.0
15.....	9	53	A.M.	rise after =1.8	rise after =2.0	rise after =1.0
15.....	9	56		fall after =1.7	fall after =1.8	fall after =1.0
15.....	9	35		rise after =2.1	rise after =2.0	rise after
April 9.....	11	14½		rise after =3.1	rise after =2.0	rise after
10.....	4	21		rise after =3.0	rise after =2.0	rise after
10.....	4	23		fall after =2.7	fall after =2.1	fall after =1.0
10.....	6	32½		fall before =3.0	fall before =1.9	fall before =1.0
June 23.....	3	25		fall after =1.4	fall after =2.0	fall after
23.....	3	29		rise after =1.5	rise after =2.0	rise after
23.....	3	33		fall after =1.6	fall after =2.0	fall after
23.....	3	37		rise after =1.6	rise after =2.0	indistinct

TABLE V. (continued).

Date.	Greenwich Mean Time.		Character and size of change (vertical-force change = unity in each instance).			
			Declination.	Horizontal force.	Vertical force.	
Oct. 28.....	h m					
	10 59 $\frac{1}{2}$		hollow =2.0	hollow =2.0	hollow	
	11 27 $\frac{1}{2}$		fall after =2.1	fall after =2.1	fall after =1.0	
	11 42	A.M.	fall after =2.0	fall after =2.0	fall after	
28.....	1 8 $\frac{1}{2}$	P.M.	rise after =2.1	rise after =2.0	rise after	
Dec. 4.....	11 18	A.M.	rise after =2.0	rise after =2.0	rise after	
	8 12		fall before =2.2	fall before =2.1	fall before =1.0	
	11 8 $\frac{1}{2}$		fall after =2.0	fall after =2.0	fall after	
	11 11 $\frac{1}{2}$	A.M.	rise after =2.0	rise after =2.0	indistinct	
1859.						
Feb. 26.....	12 53 $\frac{1}{2}$	P.M.	fall after =2.1	fall after =2.0	fall after	
	1 59 $\frac{1}{2}$	P.M.	peak =2.2	peak =2.0	peak	
	3 12 $\frac{1}{2}$	A.M.	hollow =2.0	hollow =2.0	hollow	
	3 47 $\frac{1}{2}$	A.M.	peak =2.2	peak =2.0	peak	
	3 8 $\frac{1}{2}$	P.M.	fall after =2.2	fall after =1.9	fall after =1.0	
	3 12 $\frac{1}{2}$		rise after =2.7	rise after =2.2	rise after =1.0	
June 8.....	11 43 $\frac{1}{2}$		rise after =2.5	rise after =2.2	rise after =1.0	
	11 59 $\frac{1}{2}$	P.M.	peak =2.3	peak =2.0	peak	
	11 7 $\frac{1}{2}$	A.M.	fall after =2.1	fall after =2.1	fall after =1.0	
Sept. 1.....	11 16	A.M.	<i>fall after</i> =1.4	rise after =1.9	rise after =1.0	
	2 14	P.M.	rise after =1.2	rise after =2.2	rise after =1.0	
Oct. 17.....	11 42	P.M.	fall after =3.0	fall after =2.0	fall after =1.0	
	12 31	A.M.	fall after =3.2	fall after =2.0	fall after =1.0	
	12 39		fall after =3.1	fall after =2.1	fall after =1.0	
	8 21	A.M.	fall after =2.6	fall after =2.0	fall after	
Dec. 14.....	12 7 $\frac{1}{2}$	P.M.	rise after =2.4	rise after =2.0	rise after	
	12 18	P.M.	rise after =2.5	rise after =2.0	rise after =1.0	
1860.						
April 10.....	5 15 $\frac{1}{2}$	A.M.	rise after =2.7	rise after =1.9	rise after =1.0	
	5 20 $\frac{1}{2}$		fall after =3.2	fall after =2.0	fall after	
	5 22		rise after =2.9	rise after =2.0	rise after =1.0	
	10.....	6 3		fall after =3.3	fall after =2.0	fall after =1.0
	10.....	6 7	A.M.	rise after =3.3	rise after =2.0	rise after =1.0
	13.....	12 54 $\frac{1}{2}$	P.M.	rise after =2.9	rise after =1.9	rise after =1.0
July 5.....	3 20	A.M.	fall after =3.6	fall after =2.1	fall after =1.0	
	4 34		fall after =3.5	fall after =2.1	fall after =1.0	
	7 21 $\frac{1}{2}$		fall after =2.8	fall after =1.7	fall after =1.0	
	8 10	A.M.	peak =3.4	peak =2.1	peak =1.0	
Aug. 10.....	6 57 $\frac{1}{2}$	P.M.	fall after =1.1	fall after =2.2	fall after =1.0	
	7 1 $\frac{1}{2}$		rise after =1.2	rise after =2.2	rise after =1.0	
	11.....	4 18		fall after =1.1	fall after =2.0	fall after =1.0
	11.....	4 21		rise after =0.9	rise after =1.7	rise after =1.0
	11.....	4 25 $\frac{1}{2}$		fall after =0.8	fall after =2.1	fall after =1.0
	11.....	4 29		rise after =1.1	rise after =2.0	rise after =1.0
	11.....	4 31		fall after =1.1	fall after =1.8	fall after =1.0
	11.....	4 36 $\frac{1}{2}$		rise after =1.0	rise after =2.1	rise after =1.0
	11.....	4 39		fall after =1.1	fall after =1.8	fall after =1.0
	11.....	4 41	P.M.	rise after =1.0	rise after =1.7	rise after =1.0

27. It appears from this Table that the proportional value of the declination-element is very nearly constant during a disturbance, but that it varies much from one disturbance to another. The constancy of this element during the same disturbance will perhaps be best seen by comparing the values in the declination column with those in the column of the horizontal force, as the vertical-force changes being small, any error of measurement is very apt to alter the proportion between them and the larger changes with which they are compared.

28. It will also be noticed that the great disturbance of August—September 1859 seems to consist of two disturbances superimposed, one being of the normal type, but the other, as regards the declination-element, being decidedly abnormal. This agrees well with the results of Table III.

29. It would thus appear that in the great majority of cases only one type or group of forces operates in producing a disturbance, and that the various individual forces which compose this group have a very small range as regards their mode of action upon the three elements of the earth's magnetism at Kew. We may therefore (approximately at least) represent a disturbance by a single force; and for this purpose it is competent for us to average for each disturbance the results of Table V.

30. Now that force which affects the declination is evidently the horizontal component of the disturbing magnetic force which acts in a direction perpendicular to the magnetic meridian, and its value will be $X \tan \delta\theta$, where X denotes the absolute horizontal force, and $\delta\theta$ the angular displacement of the declination magnet. But an inch of the declination-ordinate represents $22'$, hence the disturbing force which would cause a change to this amount will be $=3.8 \tan 22' = .024$, while, as we have already seen, an inch of change in the horizontal-force ordinate denotes a disturbing force $=.038$. Again, decreasing ordinates represent increasing westerly declination; and since (except in one case) the three curves are simultaneously affected in the same direction, it follows that (with the same exception) a magnetic force acting from magnetic south to north must be compounded with one acting from magnetic east to west, and with one acting vertically downwards. But, for the anomalous disturbance of August—September 1859, a force acting from magnetic south to north must be compounded with one acting from magnetic west to east.

Applying now the ordinary rules for combining forces, we obtain the following Table, in which the astronomical azimuth and dip of the various disturbing forces are given.

31. It will, however, be first necessary to allude to a peculiarity in this method of determining the direction of the disturbing force. This is, that our results apply in strictness only to those small and rapid changes which occur as it were on the surface of the great disturbance-wave.

We might, for instance, be engaged in studying the direction of that force which would occasion a small depression which we had observed to occur simultaneously in the three curves, while at the same time these curves might be elevated above their normals, and not depressed by the main body of the disturbing force. We have, however, in art. 15 given grounds for supposing that these small and rapid peaks and hollows denote

changes in the main body of the disturbing force. Our results regarding the peaks and hollows may therefore, perhaps, be viewed as applicable to the main body of the force; but we must bear in mind that while we may thus be enabled to determine the resultant line of action of this force, we do not yet learn whether it is of a positive or negative nature, whether it may be represented by a north or by a south pole. This will be rendered evident when we reflect that a small depression may be due either to the weakening of an elevating force, or to the strengthening of one of an opposite character. It thus appears that the north or south nature of the whole disturbing force must be determined by reference to the general appearance of the curve. Indeed it will be afterwards shown that we have grounds for supposing two antagonistic forces to be in operation at once. To fix our thoughts, however, let us suppose for the following Table the disturbing force to be one which increases the earth's horizontal intensity at Kew, and let us consider its action on the north pole of the needle.

TABLE VI.—Representing the Astronomical Azimuth and Dip of the various disturbing forces.

Date of disturbance.	Direction in which the force tends to make the north pole of a needle point.	Dip which the force would give to the north pole of a needle.
1858. March 14—15	North 53° 46.5' West	15° 30.5'
April 9—10	64 28.5	13 0.5
June 23	47 13	15 53
October 28.....	54 5.5	14 43.5
December 4—5	54 5.5	14 43.5
1859. February 26—27	56 27.5	14 23.5
June 8—9	56 10.5 West	13 53.5
September 1—4.....	3 27.5 East	16 46
September 1—4.....	40 30.5 West	15 11
October 17—18.....	64 21.5	12 52.5
December 14	59 13.5	14 1.5
1860. April 10—13	65 54	12 55
July 5.....	67 54	12 17
August 10—11	North 40 1.5 West	16 59.5

32. It has been already mentioned that we do not ascertain by our method whether the whole disturbing force is positive or negative. The following considerations, however, may serve to elucidate this subject.

We have seen in art. 18 that a disturbance cannot be represented by the action of a single force, and we have also seen (art. 29) that the various forces which compose a disturbing group have a very small range as regards their mode of action upon the three elements, this range being especially small when the two force-elements are compared together.

We might therefore hope to find, on inspecting the general appearance of the curves, that the two force-elements are simultaneously raised or depressed with reference to their normals nearly in the proportion denoted by the nature of the disturbing force whose character we have been analysing.

But this is not often the case; and although generally both elements are on the same side of their normals, yet it happens occasionally that the one element is above while the other is below; while on such occasions there is nothing in the behaviour of the peaks and hollows to indicate the action of any other than the ordinary disturbing force.

33. This may perhaps be explained by supposing two antagonistic forces to be in operation at once, the one tending to elevate, and the other to depress the curve, the absolute values of these forces bearing a somewhat large proportion to their differences, and the one force affecting the elements in a slightly different manner from the other.

Suppose, for instance, that $+1$ and -1 denote the position of the horizontal and the vertical-force curves at the same moment, the former being elevated above its normal, and the latter depressed below it.

Conceive this result due to the action of an elevating force represented by $+40 +20$ opposed by a depressing one represented by $-39 -21$, and we have a sufficient explanation of this anomalous circumstance, while at the same time both the antagonistic forces sufficiently present the normal type.

34. The same style of reasoning will apply in comparing the declination curve with that of either force; only here we must suppose that the one force affects the declination in a somewhat different manner from its antagonist, so that the proportion between a declination peak and one of either force is not so constant as that between peaks of the two forces. The idea of two antagonistic forces, the difference of which represents the visible action, seems also to give the simplest explanation of the fact, that sometimes a force which depresses one of the elements will change in the course of a few hours to one which elevates it—since otherwise we must suppose the disturbing force to have changed its character.

This also is the view of disturbing forces which General SABINE, who has studied the subject so long and so successfully, has lately been disposed to adopt on other grounds; and I am happy to think that the idea herein advocated is that of one whose judgment is so mature and whose information is so extensive.

35. Adopting this view of the subject, it is worthy of remark that the same element of the disturbing forces (the declination) which changes so much its comparative value from one disturbance to another, changes also the most of the three when we pass from the one to the other of the antagonistic forces concerned in the same disturbance.

In conclusion, I may be allowed to state that this paper is submitted to the Royal Society rather as indicating a method of analysis than as embodying the results of an investigation.

Note on the Electromotive Force induced in the Earth's Crust by Variations of Terrestrial Magnetism. By Prof. W. THOMSON, A.M., L.L.D., F.R.S.

The evidence from observation adduced in the preceding paper tending to show that some "earth-currents" which have been actually observed have been the electro-magnetically induced effects of variations of terrestrial magnetism, appears to be a very important contribution to the discovery of the complete theory of these most interesting and perplexing phenomena. It necessarily, however, suggests the question, Is the electromotive force induced by variations of terrestrial magnetism comparable in amount with that which is found in observations on earth-currents? There is scarcely occasion at present for working out a complete mathematical theory of the currents induced in the earth's crust by any fully specified magnetic variations. This can easily be done as soon as observation supplies data enough to make it useful. In the mean time a very rough theoretical estimate of the absolute amount of electromotive force induced by such magnetic variations as we know to exist, is sufficient to render it certain that this electro-magnetic induction does very sensibly influence the observed phenomena of earth-currents. For if there be, as we know there is every day, a gradual variation of terrestrial magnetism over a large portion of the earth's surface, amounting to a minute of angle in the direction of the dipping-needle, or to some thousandths, or not much less than one thousandth of force during several hours, this change of magnetism must induce in a length of a few hundred miles in the earth's crust an electromotive force, which we readily see must be comparable with that which would be induced in a linear conductor of the same length if carried across the lines of magnetic force at the rate of a minute of arc (or a nautical mile) in several hours. In two articles communicated eleven years ago to the 'Philosophical Magazine'*, I explained the principles on which such an electromotive force as this is to be compared with familiar standards, as, for instance, that of an element of DANIELL'S battery. Thus a horizontal conductor, 1,400,000 feet long (or about 270 British statute miles), carried at the rate of 600 feet (or about one-tenth of a minute of arc) per hour in a horizontal direction perpendicular to its own length across the British Isles or neighbouring Atlantic ocean (where the vertical magnetic force averages about 10 British absolute units of magnetic force), would experience an induced electromotive force amounting to $\frac{600}{3600} \times 10 \times 1,400,000$, or about 2,300,000 British absolute units of electromotive force. But, as I showed in the second of those articles, the electromotive force of a single cell of DANIELL'S is about 2,500,000 British absolute units. Hence the induced electro-magnetic force in question is about equal to that of a single cell. Some such electromotive force as this, therefore, must be induced in a length of 270 miles of the earth's crust, by the ordinary diurnal variations of terrestrial magnetism; and the much more rapid variations in magnetic storms

* See 1851, Second half year. "On the Mechanical Theory of Electrolysis," and "Applications of the Principle of Mechanical Effect to the Measurement of Electromotive Forces, and of Galvanic Resistances, in Absolute Units."

must produce much greater electromotive forces, which we may conceive may not unfrequently be as much as that of ten or twenty cells, and sometimes may amount to 100 cells or more. Just such amounts of electromotive force were those which I actually observed in the Atlantic cable, as the following extract from the 'Encyclopædia Metropolitana,' article "Telegraph, electric," shows.

"In the failure of the Atlantic Cable in September 1858, the portion terminating at Valencia came to give nearly the same indications as an insulated conductor about 270 miles long, laid out westward, and connected with a copper plate sunk at a little less than that distance in the Atlantic. In these circumstances the writer found that from 1 to 9 or 10 twentieths of the electromotive force of two DANIELL'S elements was generally sufficient to balance the earth-current; not unfrequently 14 or 15 were required; sometimes, although rarely, 20, or the full electromotive force of two DANIELL'S elements, was insufficient; and once or twice in the course of the month of September, earth-currents were received so strong that five or six DANIELL'S elements would have been required to balance them."

It seems therefore quite certain that the ordinary every-day earth-currents in that locality must be very sensibly influenced by electro-magnetic induction from the ordinary diurnal variations of terrestrial magnetism; but it is also quite certain that they are only in part due to this cause, and that some more powerful, but as yet unknown agency, is at work to produce them. For although I found that a day seldom, if ever, passes without the direction of the current changing several times, yet there was no relation between the times of such changes and the solar hours. I conclude with the following additional extract from the same article, expressing views regarding earth-currents which I think will be found to agree with the extensive and careful observations of Mr. C. V. WALKER, which have been published since it was written, although they seem quite at variance with the theory which has recently been advocated by Prof. LAMONT and Dr. LLOYD, that earth-currents, however they are themselves generated, do directly produce the magnetic variations.

"Earth-currents are certainly related to the irregular variations of terrestrial magnetism, since they are always found unusually strong during brilliant displays of aurora borealis; for it has long been known that, on these occasions, the magnetic disturbances are unusually strong. Being related to the variations of terrestrial magnetism, it is probable that the earth-currents also will be found to have daily periods; but, in the mean time, we only know that, while the diurnal variation in terrestrial magnetism is observable in general every day, and is only on rare occasions overborne by irregular disturbances, the earth-currents vary each day from hour to hour, like the wind, under some overpowering non-periodic influence, and can only show daily periodicity in residual averages derived from lengthened series of observations. It is probable that careful synchronous observations of auroras, earth-currents, and variations of terrestrial magnetism, will lead to a discovery of the primary influence, whether in the earth, or terrestrial atmosphere, or surrounding interplanetary air, which causes these phenomena."—W. T.